SPECTRA-SOLARIS Inc.

Theoretical Approach

Project 526814 – Feasibility Study of Using Electrical Noise in Solar Cells as a Quality Indicator
(abbreviated version)

1. The Contact Noise on Thermodynamic Equilibrium Conditions

1.1 Noise generation mechanism

On equilibrium, two opposite and equal currents are flowing across the potential barrier. Those two currents are canceling each other in a statistical sense, since accordingly to the Second Principle of Thermodynamics the net current must be zero.
The barrier potential difference is maintained by those two competing currents.
If both types of charge carriers (electrons and holes) are present this statistical balance will be achieved for both of them, independently. In other words, there are two electron currents canceling each other and two hole currents canceling each other.
This assertion is true regardless of the nature of the potential barrier.

Admitting, for simplification, that there are no collisions between charge carriers and the atomic structure of the material, these currents across the barrier will generate a specific electrical noise.

Considering a very general structure with a potential barrier, the respective currents across the barrier will be J_{12} and J_{21} (Figure 1.1).

Figure 1.1 The currents across the potential barrier

The net current across the barrier will be:

$$J = J_{12} - J_{21} \quad (1.1)$$

At thermodynamic equilibrium, the net macroscopic current is zero:

$$J \equiv 0 \quad (1.2)$$

Therefore:

$$J_{12} = J_{21} \equiv J_0 \quad (1.3)$$

where J_0 is the saturation current.

Considering the statistical canceling of currents, this in Langevin method can be written in a form to conserve the total electrical charge:

$$j_{12}(t) + \chi_1(t) = j_{21}(t) + \chi_2(t) \tag{1.4}$$

where $j_{12}(t)$ and $j_{21}(t)$ are the instantaneous values of the respective currents and $\chi_1(t)$ and $\chi_2(t)$ are two random functions, characteristic to the transit process across the barrier.

Integrating (1.4) on a time interval many times longer than the duration of the relaxation processes, will result:

$$< j_{12}(t) > + < \chi_1(t) >=< j_{21}(t) > + < \chi_2(t) > \tag{1.5}$$

But:

$$\begin{aligned} &< j_{12}(t) >= J_{12} \\ &< j_{21}(t) >= J_{21} \end{aligned} \tag{1.6}$$

Therefore:

$$< \chi_1(t) >=< \chi_2(t) > \tag{1.7}$$

because the equality of the averages does not have to depend on integration time (if long enough).

In conclusion the microscopic statistical balancing principle leads to the important conclusion of total correlation of charge carrier fluctuations in the left and in the right side of the barrier (considering the barrier transit process only and neither the generation-recombination nor capture processes). The consequence is that it is enough to calculate the charge carrier density fluctuations on one side of the potential barrier, only.

The electrical fluctuations associated with those barrier crossings are due to the discrete nature of the transported electrical charges. The current can be considered a superposition of elementary Dirac pulses with a weight of q:

$$J(t) = q\sum_j \delta(t - t_j) \tag{1.8}$$

where t_j is the moment of time when the respective q charge starts its transit across the barrier (obviously the t_j moments are part of a random suite).

As a result, each current J_{12} and J_{21} will generate the squared mean noise current (shot noise) given by:

$$< i^2 >= 2q|J| \cdot df \tag{1.9}$$

As a conclusion the whole structure will generate the total squared mean noise current:

$$< i^2 >= 2q|J_{12}| \cdot df + 2q|J_{21}| \cdot df = 2q\left[|J_{12}| + |J_{21}|\right] \cdot df \tag{1.10}$$

At thermodynamic equilibrium:

$$< {i_0}^2 >= 4qJ_0 \cdot df \tag{1.11}$$

The power spectral density (PSD) will be:

$$S_{J_0}(f) = 4qJ_0 \tag{1.12}$$

therefore a *white spectrum*.

In reality, considering the transit duration across the barrier, τ_t, the spectrum has the form:

$$S_{J_0}(f) = 4qJ_0\left[\frac{\sin(\pi f \tau_t)}{\pi f \tau_t}\right]^2 \approx 4qJ_0 \frac{1}{1+\frac{\tau_t^2\omega^2}{12}} \tag{1.13}$$

which has the cut-off frequency:

$$\omega \cong 3.5 / \tau_t$$

If collisions are present during the transit (with a relaxation time of τ_r), the shot noise appears only if $\tau_r > \tau_t$ and the spectrum becomes:

$$S_{J_0}(f) = 4qJ_0 \cdot \exp(-\tau_t / \tau_r) \tag{1.14}$$

Here $\exp(-\tau_t / \tau_r)$ is the collision free barrier crossing probability.

The normal situation is to consider frequencies far less than $1/\tau$ and to use (1.12).

1.2 Barrier Contact Resistance at Zero Bias

For a very convenient parameter to characterize a potential barrier we can introduce a *barrier contact resistance* at zero bias.
This can be defined as:

$$R_c = \left(\frac{\partial J}{\partial V}\right)^{-1}_{V=0} \tag{1.15}$$

The unit of measure is Ohm/cm^2.

A good contact is characterized by a minimum contact resistance.

For a given barrier, R_c depends on barrier height and on charge carrier concentrations in both regions in contact.

As a consequence one can establish an equivalent noise circuit for this structure in equilibrium conditions, neglecting for this moment the resistance of the regions far from barrier (they do not participate on contact phenomena).

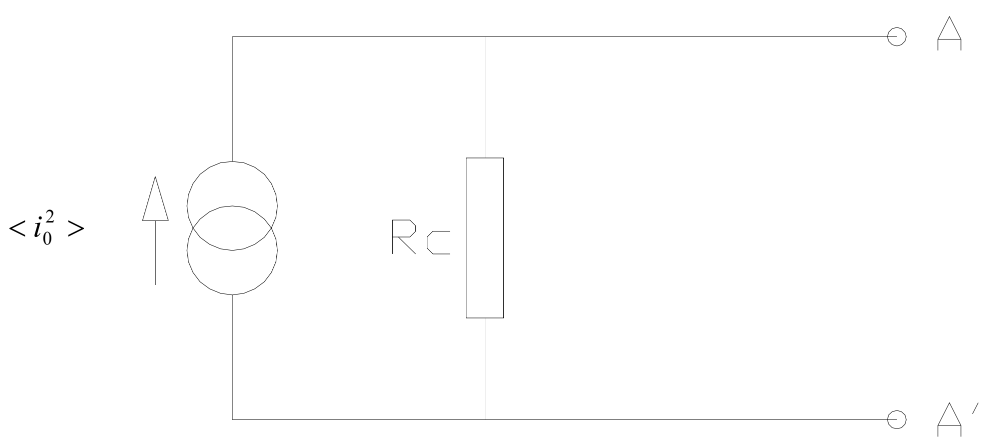


Figure 1.2 Equivalent noise circuit at zero bias

The squared mean noise voltage present at terminals A and A' is going to be:

$$< e_0^2 > = R_c^2 < i_0^2 > \tag{1.16}$$

with power spectral density as:

$$S_{e_0}(f) = R_c^2 \cdot S_{J_0}(f) \tag{1.17}$$

In order to obtain (1.17) we used the fact that if a random signal *X(t)* is at the input of a linear system with the transfer function *Y(f)* and at its output is *Z(t)*, the respective DSPs being $S_X(f)$ and $S_Z(f)$, then:

$$S_Z(f) = S_X(f) \cdot |Y(f)|^2 \tag{1.18}$$

The linearity approximation is correct since the electrical noise has a rms value by far less than *kT/q* (even at very low temperatures), which intervene in the normal I-V contact equation, as we are going to see later.

Therefore:

$$S_{e_0}(f) = 4qJ_0 \left[\left(\frac{\partial J}{\partial V} \right)_{V=0} \right]^2 \tag{1.19}$$

This is a very general relation that can be applied at every potential barrier, regardless of its nature.

This relation is important because the squared mean noise voltage:

$$< e_0^2 > = \int S_{e_0}(f) \cdot df$$

is a directly measurable parameter that can give significant information about the potential barrier at equilibrium, without disturbing the barrier. It is worthy to mention that this is a true equilibrium method. Other methods normally extrapolate the non-equilibrium values to zero bias. If hysteresis or non-linear behavior is present, those methods will lead to significant errors.

The very general I-V characteristic of a potential barrier has the expression:

$$J = J_0 \left[\exp\left(\frac{qV}{nkT} \right) - 1 \right] \tag{1.20}$$

where *k* is the Boltzmann's constant, *T* the absolute temperature, and *n* the ideality coefficient with values ranging from 1 to 2.

Considering (1.15) and (1.20):

$$R_c = \frac{nkT}{qJ_0} \tag{1.21}$$

and from (1.19) and (1.21):

$$S_{e_0}(f) = 4kTnR_c \tag{1.22}$$

which for $n=1$ returns the well know Nyquist relation giving the PSD of thermal noise generated by a pure resistance of value R_c.

This relation can be applied as well for small biases ($V<< kT/q$), the noise being even in this case generated by Rc.

For $n \neq 1$ the Nyquist theorem is not valid any longer (in the transit mechanism across the barrier the speed distribution of charge carrier is not important), and PSD is n times higher than the noise given by Rc.

An equivalent noise resistance can be defined as:

$$R_{eq} = n \cdot R_c \tag{1.23}$$

which for $n=1$ gives:

$$R_{eq} = R_c \tag{1.24}$$

As a consequence a non-ideal barrier will have an electrical noise 6dB higher than a normal pure resistance of the same value.

1.3 Barrier Capacitance Influence

As it is known, in the contact region, the spatial electrical charge is modified due to the transit process across the barrier. This can be equivalent with a plan capacitor with the electrodes equaling the width of the zone with charge modification. The capacitance depends on contacting materials and on external applied voltage. Considering zero bias, this equivalent capacitance is Cc (per area unit) and the equivalent circuit will be:

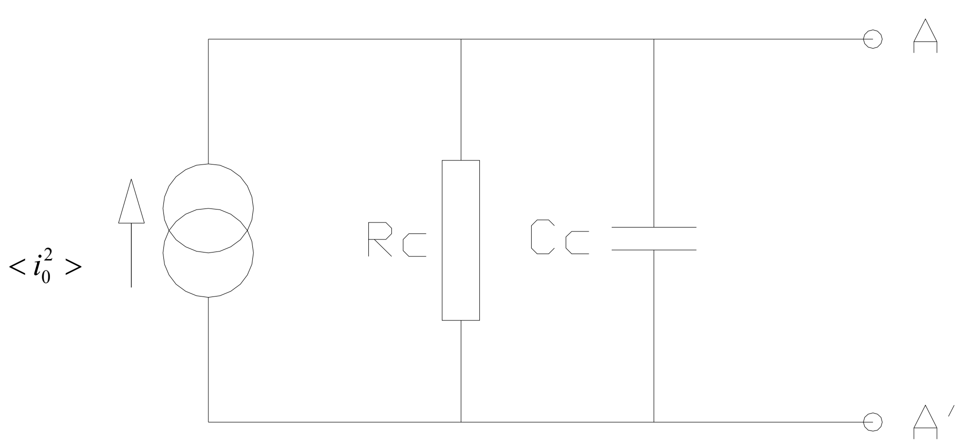


Figure 1.3 Equivalent noise circuit at zero bias considering the barrier capacitance
The squared mean noise voltage PSD at A, A' terminals will be:

$$S_{u_0}(f) = S_{J_0}(f) \cdot \left|Y(f)\right|^2 \tag{1.25}$$

Here $\left|Y(f)\right|$ is the module of the circuit transfer function given by:

$$\left|Y(f)\right|^2 = \frac{R_c^2}{1 + (2\pi f \cdot C_c R_c)^2} \tag{1.26}$$

Therefore:

$$S_{u_0} = 4qJ_0 \frac{R_c^2}{1 + (2\pi f \cdot \tau_c)^2} \tag{1.27}$$

where $\tau_c = R_c \cdot C_c$ is the *time constant* of the contact.

Considering (1.22):

$$S_{u_0}(f) = \frac{S_{e_0}(f)}{1 + (2\pi f \cdot \tau_c)^2} \tag{1.28}$$

The noise spectrum is not white any longer.

It is evident then that from noise measurements at zero bias one can obtain R_c, C_c, J_0 and n that characterize the potential barrier.

2. The Contact Noise on Non-Equilibrium Conditions

The most common approach to non-equilibrium noise problem is to address in detail the transit processes across the barrier for different particular situations.

In the following, a more general method, based on thermodynamic arguments will be developed.

Considering the Nyquist theorem and the so-called fluctuation-dissipation theorem (Callen) there is a interdependence between the generated noise and power dissipation in a certain device. This interdependence is obtained through a generalized dissipative function (electrical resistance in general sense).

The dissipated power, P, in a pure resistor of value R_e, when a voltage V is applied across it, is given by:

$$P = \frac{V^2}{R_e} \qquad (2.1)$$

A small voltage fluctuation, dV, superposed over V will generate an extra absorbed power, dP, obtained by differentiating (2.1):

$$dP = \frac{2V}{R_e} \cdot dV \qquad (2.2)$$

It was considered that for a small variation dV, the resistance Re has a linear behavior.

Therefore:

$$R_e = 2V\left(\frac{dP}{dV}\right)^{-1} \qquad (2.3)$$

This very general relation can associate an equivalent noise resistance R_e to any device if one can estimate de power variation, dP, associated to a voltage fluctuation, dV at its terminals.

For a contact process:

$$J = J_0\left(\exp\left(\frac{qV}{kT}\right) - 1\right) \qquad (2.4)$$

the dissipated power will be:

$$P = J \cdot V = J_0 \cdot V\left(\exp\left(\frac{qV}{kT}\right) - 1\right) \tag{2.5}$$

and differentiating:

$$\frac{dP}{dV} = J_0\left(\exp\left(\frac{qV}{kT}\right) - 1\right) + J_0\frac{qV}{kT}\exp\left(\frac{qV}{kT}\right) = J + \frac{qV}{kT}(J + J_0) \tag{2.6}$$

From (2.2) and (2.3):

$$R_e = \frac{2V}{J + \frac{qV}{kT}(J + J_0)} \tag{2.7}$$

The differential conductance of the contact is given by:

$$G_d = \frac{1}{R_d} = \frac{\delta J}{\delta V} = \frac{J_0 q}{kT}\exp\left(\frac{qV}{kT}\right) = \frac{q}{kT}(J + J_0) \tag{2.8}$$

and the constant current conductance:

$$G_{cc} = \frac{1}{R_{cc}} = \frac{J}{V} = \frac{Jq}{kT \cdot \ln\left(1 + \frac{J}{J_0}\right)} \tag{2.9}$$

Considering (2.7), (2.8) and (2.9) and introducing the noise equivalent conductance as $G_e = 1/R_e$, results:

$$G_e = \frac{G_{cc} + G_d}{2} \tag{2.10}$$

This is a very interesting relationship that shows that the thermal nose equivalent conductance for each contact process is the arithmetic media between the differential and cc conductance.

In general G_e depends on J since G_{cc} and G_d, both depend on J (deviation from equilibrium).

At zero bias, J=0 and then:

$$G_{e_0} = G_{d_0} = G_{cc_0} = \frac{1}{R_c} = \frac{qJ_0}{kT} \tag{2.11}$$

This verifies that at zero bias, Rc is the noise equivalent resistance.

The squared mean noise current PSD at terminals will be:

$$S_J(f) = 4kT \cdot G_e = 2kT(G_{cc} + G_d) \tag{2.12}$$

and considering (2.8) and (2.9):

$$S_J(f) = 2q(J + J_0) + \frac{2qJ}{\ln\left(1 + \frac{J}{J_0}\right)} \tag{2.13}$$

A simplified formula for a p-n junction considering in detail the mechanism of transport across the barrier was obtained as:

$S_J(f) = 2q(J + J_0) + 2qJ_0$ which is valid just for $J/J_0 << 1$, therefore a particular case for (2.13).

The squared mean noise voltage PSD at terminals will be:

$$S_e(f) = 4kT\frac{1}{G_e} = 4kTR_e = \frac{8kT}{G_{cc} + G_d} = 8kT\frac{R_{cc} \cdot R_d}{R_{cc} + R_d} \tag{2.14}$$

It is easy to verify that at zero bias (*V=0*):

$$\begin{aligned} S_J(f)\big|_{V=0} &= 4qJ_0 \equiv S_{J_0}(f) \\ Se(f)\big|_{V=0} &= 4qTR_c \equiv S_{e_0}(f) \end{aligned} \tag{2.15}$$

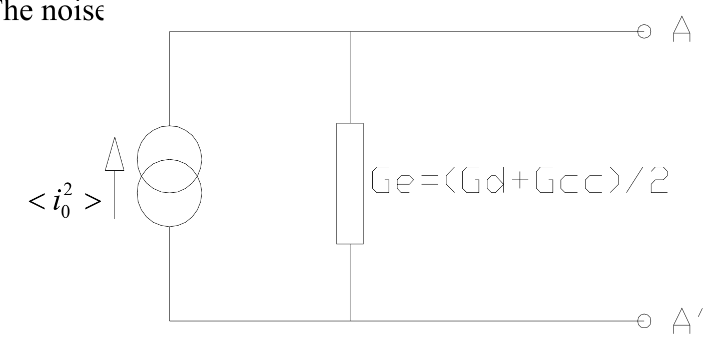
with:

$$R_c = \frac{kT}{qJ_0} \equiv R_d\big|_{V=0} \equiv R_{cc}\big|_{V=0}$$

because:

$$\ln(1+x) = \frac{x}{1} - \frac{x^2}{2} + \frac{x^3}{3} + ... \approx x \quad \text{if } x << 1$$

The noise



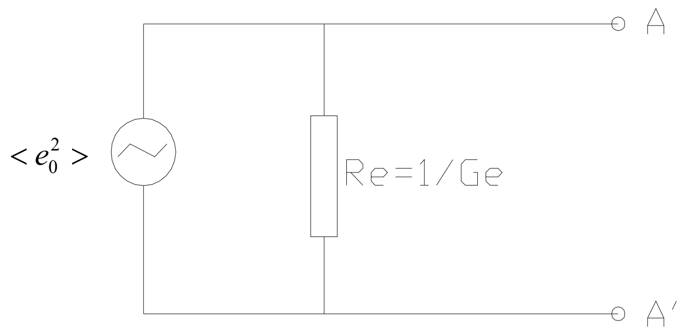


Figure 2.1 Equivalent noise circuits at non-zero bias

For :

$J >> J_0$, $J \cong J_0 \cdot \exp\left(\frac{qV}{kT}\right)$, then:

$$\frac{G_{cc}}{G_d} = \frac{\dfrac{qJ}{kT \cdot \ln\left(\dfrac{J}{J_0}\right)}}{\dfrac{qJ}{kT}} = \frac{1}{\ln\left(\dfrac{J}{J_0}\right)} << 1 \qquad (2.16)$$

Therefore:

$$G_e \approx \frac{G_d}{2} \qquad (2.17)$$

And:

$$S_J(f) \cong 2qJ = 4kT \cdot \frac{G_d}{2} \qquad (2.18)$$

$$S_e(f) \cong 4kT(2R_d) = \frac{8k^2T^2}{qJ}$$

In conclusion, at high currents the equivalent noise resistance is approximately $2R_d$.

Assuming that the pure thermal noise given by the structure is due exclusively to the differential resistance R_d, on can obtain:

$$S_e(f) \cong 4kT \cdot R_e = 4kT \cdot R_d + 4kT \cdot R_{ex} \tag{2.19}$$

where R_{ex} is the excess noise resistance.

Considering (2.10) and (2.19):

$$R_{ex} = \frac{R_d(R_{cc} - R_d)}{R_{cc} + R_d} \tag{2.20}$$

The noise ratio, *NR*, will be given by:

$$NR = \frac{4kT \cdot R_d + 4kT \cdot R_{ex}}{4kT \cdot R_d} = 1 + \frac{R_{cc} - R_d}{R_{cc} + R_d} = 1 + \frac{G_d - G_{cc}}{G_d + G_{cc}} \tag{2.21}$$

Considering (2.11) and (2.16):

if $J >> J_0$ then $NR \approx 2$ (2.22)
$J << J_0$ then $NR \approx 1$

It is obvious the contact capacitance must be considered as well; the situation is more complex since it depends on bias (through the width of the spatial charge region).
The noise equivalent circuit valid for every bias (zero bias included) but in the absence of 1/f and pulse noise is:

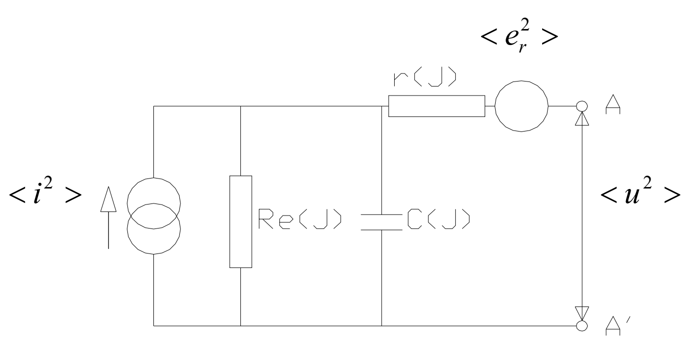


Figure 2.2 Equivalent noise circuits of the structure with a potential barrier
Here *r(J)* is the resistance of the neutral regions which are not involved in the contact phenomena.

The squared mean noise voltage PSD at terminals will be given by:

$$S_u(f) = S_J(f)\frac{R_e^2(f)}{1+(2\pi f\cdot\tau)^2} + 4kT\cdot r(J) \tag{2.23}$$

or:

$$S_u(f) = \frac{4kT\cdot R_e(J)}{1+(2\pi f\cdot\tau)^2} + 4kT\cdot r(J) \tag{2.24}$$

where:

$$\tau = R_e(J)\cdot C(J) \tag{2.25}$$

3. Solar Cell Characteristics and Efficiency Losses

5. Metal-Semiconductor Contact Noise

As is known, there are several major contributors to the series resistance of a solar cell. They are the bulk resistance of the semiconductor, the bulk resistance of the metallic contacts and interconnections and the *contact resistance* between the metallic contacts and the semiconductor. The bulk resistances are quite easily controlled. The latest component is the most difficult to control and therefore the noise aspects related to it can be quite important for a better understanding and minimizing.

Let us consider a contact metal/n-type semiconductor (M/N),
The contact potential barrier depends on the ratio between the extraction potentials of the metal, φ_m and semiconductor, φ_s.
If $\varphi_m = \varphi_s$, the structure is in equilibrium from the very beginning (static), there is not potential barrier and the contact is neutral.
If $\varphi_m > \varphi_s$, the contact is blocking and if $\varphi_m < \varphi_s$, the contact is ohmic. In practice both situations are important.

The energy diagram of the M/N structure with a blocking contact, at thermal equilibrium (Schottky) can be seen in figure 5.1.

Figure 5.1 The energy diagram of the M/N contact.

It is obvious that the spatial charge region (depletion) exists in semiconductor only, the resistivity of the metal being orders of magnitude smaller.
In figure 5.1, χ is the electron affinity of the semiconductor, $\Delta\varphi_m$ is the decrease of the barrier height produced by the image force and V_o is the contact potential difference. NF is the Fermi level that is the same in both (thermal equilibrium).

In conclusion, the potential barrier height is:

$$\varphi_B = \varphi_m - \chi - \Delta\varphi_m \qquad (5.1)$$

Considering the so called depletion approximation (in the spatial charge region is only important the positive charge of the ionized donors), the width w_o of the depleted region (potential barrier):

$$w_o = \sqrt{\frac{2\varepsilon\varphi_B}{qN_D}} \qquad (5.2)$$

where ε is the semiconductor permittivity.

The net current across the M/N barrier is given by:

$$J = J_{MS} - J_{SM} \tag{5.3}$$

where J is the overall current density and J_{SM} current density from semiconductor to metal and J_{MS} the opposite.

At equilibrium J=0 and then the saturation current becomes:

$$J_0 = J_{MS} = J_{SM} \tag{5.4}$$

Generalizing for an external applied voltage V, the diode theory gives or J_{SM}:

$$J_{SM}(V) = \frac{A^* T}{k}\int_0^{\infty} P(\xi)\exp\left(\frac{-(q\varphi_B - qV + \xi)}{kT}\right)\cdot d\xi + \frac{A^* T}{k}\int_0^{q(V_0 - V - \Delta\varphi_m)} F_S \cdot P(\eta)(1 - F_M)\cdot d\eta \tag{5.5}$$

Here A* is Richardson's effective constant, given by:

$$A^* = \frac{4\pi q m^* (kT)^2}{h^2} \tag{5.6}$$

with m* the effective mass of electron in semiconductor.

F_S and F_M are the Fermi-Dirac distribution functions in semiconductor and metal, respectively, P(ξ) and P(η) are the quantic transmission functions above and under the barrier maximum.

In (5.5) the first integral represents the thermal emission of electrons above the barrier and the second the tunneling component. Their ratio depends on barrier height, on temperature and on doping concentrations.
The same is true for J_{MS}.

$$J_{MS}(V) = \frac{A^* T}{k}\int_0^{\infty} P(\xi)\exp\left(\frac{-(q\varphi_B + \xi)}{kT}\right)\cdot d\xi + \frac{A^* T}{k}\int_0^{q(V_0 - V - \Delta\varphi_m)} F_M \cdot P(\eta)(1 - F_S)\cdot d\eta \tag{5.7}$$

There are two situations

i) Predominant thermoionic emission (Schottky)

For low doped semiconductors, the thermoionic emission dominated the tunnel effect at normal temperatures.
Admitting P(ξ)=1, one obtains the well known Richardson equation:

$$J_{SM} = J_0 \exp(\frac{qV}{kT}) \quad (5.8)$$

with:

$$J_0 = A^* T^2 \exp(\frac{-q\varphi_B}{kT}) \quad (5.9)$$

Therefore:

$$J = J_0 \left(\exp\left(\frac{qV}{kT}\right) - 1 \right) \quad (5.10)$$

because the thermoionic emission part of J_{MS} does not depend on V (hence $J_{MS}=J_0$).

As a consequence, the contact resistance will be given by (1.21):

$$R_c = \frac{kT}{qJ_0} = \frac{4k^2}{qA^*} \exp\left(\frac{q\varphi_B}{kT}\right) \quad (5.11)$$

The DSP from (1.22), results as:

$$S_{e0}(f) = \frac{4k^2}{qA^*} \exp\left(\frac{q\varphi_B}{kT}\right) \quad (5.12)$$

It is clear that the contact noise will increase exponentially as the temperature decreases and as the barrier height increases.

This is simple to justify: if it is more difficult to cross the barrier, the relative fluctuations in the number of electrons able to cross are more significant.

In the same time the noise increases when A* decreases (in other words when m* decreases). This is less important than the contributions of φ_B and T.

As a consequence, measuring $S_{e0}(f)$ at zero bias and knowing A*, the barrier height results as:

$$\varphi_B = \frac{kT}{q} \left(D + \ln\left(A^* \cdot S_{e0}(f)\right)\right) \quad (5.13)$$

where $D=\ln(q/4k^2)=60,621$ and $k/q=8.575\text{x}10^{-5}$ J/CK

If m* is not known, hence nor A*, results:

$$\ln S_{e0}(f) = \frac{q\varphi_B}{kT} + \ln\frac{4k^2}{qA^*} \quad (5.14)$$

In consequence the relationship between $\ln S_{e0}$ and 1/T is linear.

Having experimentally this dependence, its slope will give φ_B and the interception A* , therefore m*, the effective mass of majority charge carriers in semiconductor.

Defining the temperature coefficient of the contact resistance as:

$$\gamma = -\frac{1}{R_C}\frac{\partial R_C}{\partial T} \quad (5.15)$$

from (5.11) will result:

$$\gamma = \frac{1+\frac{q\varphi_B}{kT}}{T} \quad (5.16)$$

Therefore γ can be obtained from (5.14) through φ_B.

As was shown, it is possible to obtain the contact capacitance per unit of barrier area, C_c from noise measurements.
This is:

$$C_c = \frac{\varepsilon}{w_0} \quad (5.17)$$

The barrier width w_0 results and the concentration of charge carriers in semiconductor, as:

$$N_D = \frac{2\varphi_B}{q\varepsilon}\cdot\frac{1}{C_c^2} \quad (5.18)$$

In conclusion, from a simple noise measurement at zero bias, one can obtain almost all essential parameters of the metal-semiconductor contact as: contact resistance and capacitance, temperature coefficient of contact resistance, saturation current, barrier height, barrier width for semiconductors with a doping less than $10^{18}cm^{-3}$.
Some semiconductor specific parameters like effective mass and impurity concentration can be obtained as well.

A very simple noise measurement apparatus with a provisional contact to the semiconductor wafer (Mercury, for example), with a known area, can be developed for those tests.

6. P-N Junction (abrupt, asymmetrical) Noise

The abrupt and asymmetrical p-n junction is a typical junction for superficially diffused devices and is similar in many aspects to a metal-semiconductor contact, The diode theory applies.

In the so-called depletion approximation, the reverse current through the junction is:

$$J_0 = \frac{1}{2} q \frac{n_i}{\tau} w + qD \frac{n_i^2}{C_B L} = J_{0r} + J_{0d} \tag{5.19}$$

where the first term is the due to the carrier generation in depleted region and the second is the diffusion current of the carriers from neutral regions.
The generation current depends on applied reverse voltage (through w) but the diffusion current is independent.

The direct current can be expressed as:

$$J_d = J_0 \left(\exp\left(\frac{q|V_F|}{mkT} \right) - 1 \right) \tag{5.20}$$

where m=1 if the diffusion current is dominant and m=2 if the recombination current is dominant.

Therefore the contact resistance is going to be:

$$R_C = \frac{mkT}{qJ_0} \tag{5.21}$$

There are two situations:

1. the case when m=1

The diffusion mechanism is dominant at room temperature.
Then:

$$R_C = \frac{kT}{qJ_{0d}} \tag{5.22}$$

with:

$$J_{0d} = qD_p \frac{n_i^2}{C_B L_p} = qD_p \frac{p_{n0}}{L_p} \tag{5.23}$$

for a n^+p junction. Here p_{n0} is the holes concentration in the neutral n region.

Therefore the Nyquist relation is valid and because:

$$n_i = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2} \exp\left(\frac{-\Delta w}{2kT}\right) \tag{5.24}$$

with Δw the forbidden gap width, results:

$$J_{0d} = 4\frac{qD_p}{L_p C_B}\left(\frac{2\pi m^* k}{h^2}\right)^3 T^3 \exp\left(\frac{-\Delta w}{kT}\right) = A^* T^3 \exp\left(\frac{-\Delta w}{kT}\right) \tag{5.25}$$

and

$$S_{e0}(f) = \frac{4k^2T^2}{qT^3}\exp\left(\frac{\Delta w}{kT}\right) = \frac{B}{T}\exp\left(\frac{\Delta w}{kT}\right) \tag{5.26}$$

where:

$$B = \frac{4k^2}{qA^*}$$

Therefore:

$$\ln\left(T \cdot S_{e0}(f)\right) = \ln B + \frac{\Delta w}{kT} \tag{5.27}$$

because at low concentrations D_p and L_p are practically constants.

As a consequence, from the characteristic $S_{e0}(f)$-T one can obtain Δw and C_B and next φ_B and w_0 through:

$$\varphi_B = 2\frac{kT}{q}\ln\frac{C_B}{n_i} \tag{5.28}$$

$$w_0 = \sqrt{\frac{2\varepsilon_r \varepsilon_0 \varphi_B}{qC_B}} \tag{5.29}$$

The maximum electrical field at equilibrium and the barrier capacitance will be:

$$E_{max} = 2\frac{\varphi_B}{w_0} \tag{5.30}$$

$$C_0 = \frac{\varepsilon_r \varepsilon_0}{w_0} \tag{5.31}$$

At zero bias the diffusion capacitance is important, too:

$$C_{d0} = \frac{q^2}{2kT}\left(L_p p_n + L_n n_p\right) \approx \frac{q^2}{2kT} L_p p_n \tag{5.32}$$

which can be obtained as well since $p_n = \frac{n_i^2}{C_B}$.

2. the case when m=2

In this situation the recombination current is dominant and:

$$R_C = \frac{2kT}{qJ_{0r}} \tag{5.33}$$

and still considering that the Nyquist theorem is valid (being thermal equilibrium) it is obvious that $S_{J_0}(f) = 4qJ_0$ does not apply any longer and the generation-recombination noise ceases to be white.

6. Statistical Method of Noise Measurement

The vast majority of noise processes have a Gauss-type statistical distribution of the noise amplitude.
There are two situations related to the width of the frequency band of noise measurement.

6.1 Broad Band Noise

In this situation, the probability that in absolute value the Gauss noise will be smaller than an arbitrary value $E \in (-\infty,+\infty)$ is given by:

$$P_a(E) = P(E) - P(-E) = erf\left(\frac{E - E_0}{\sigma\sqrt{2}}\right) \quad (6.1)$$

The Gauss noise has a non-zero probability to exceed any value if one can wait enough long time, indeed.

We used the well-known distribution function for Gauss-type processes:

$$P(E) = \frac{1}{2}\left(1 + erf\left(\frac{E - E_0}{\sigma\sqrt{2}}\right)\right) \quad (6.2)$$

where σ^2 is the variance and E_0 is the mean value of the random variable.

$$\begin{aligned} E_0 &= \langle E \rangle \\ \sigma^2 &= \operatorname{var} E = \langle E^2 \rangle - \langle E \rangle^2 = \langle E^2 \rangle - E_0 \end{aligned} \quad (6.3)$$

The error function, erf(x) is defined by:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{x^2} \cdot dx \quad (6.4)$$

The complementary probability, that the Gauss noise will be bigger in absolute value than E is:

$$P_{ca}(E) = 1 - P_a(E) = \left(1 - erf\left(\frac{E - E_0}{\sigma\sqrt{2}}\right)\right) = erfc\left(\frac{E - E_0}{\sigma\sqrt{2}}\right) \quad (6.5)$$

where erfc(x) is the complementary error function represented in figure 6.1.

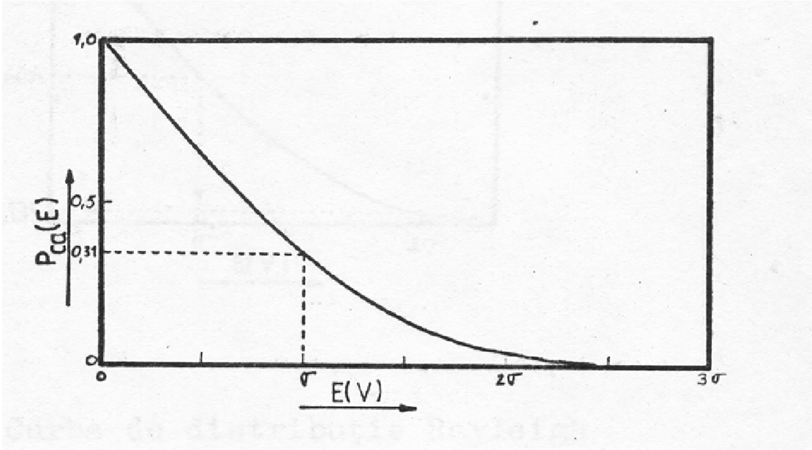


Figure 6.1 Gauss Noise Distribution Function

Admitting a null mean value, <E>=0, from (6.5) the probability to have a signal bigger than σ is:

$$P_{ca}(\sigma) = erfc\left(\frac{\sigma}{\sigma\sqrt{2}}\right) = erfc(0.707) = 0.317 \qquad (6.6)$$

It is clear than that having experimentally the distribution curve for noise amplitude, σ is the abscise corresponding to P_{ca}=0.317, hence $<E^2>$.
If <E>≠0, then $<E^2>$ results from (6.3), the abscise from Figure 6.1 becoming $E-E_0$.

Let's consider the following noise measurement circuit.

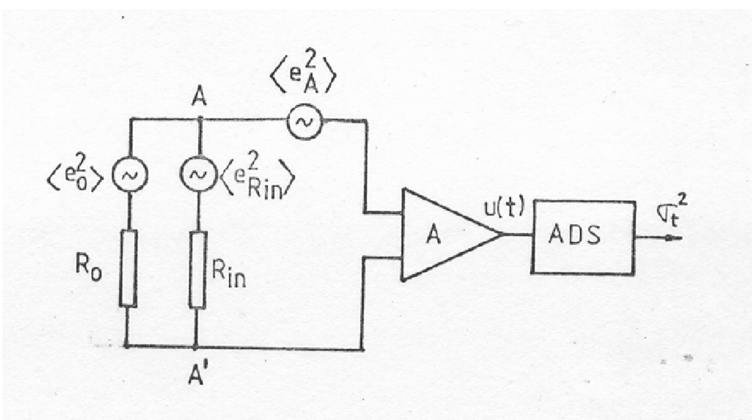


Figure 6.2 Noise Measurement Circuit by Statistical Distribution

Here A is a broad band amplifier with B_{eff} the effective noise bandwidth having the transfer function $|Y(f)|=Y_0$, adjustable but constant in the band and R_{in} its input resistance, with its equivalent noise generator $<e_{Rin}^2>$.
The device under test has the internal resistance R_0 and its equivalent noise generator $<e_0^2>$. Normally Rin>>R0 and therefore the noise is given practically by $<e_0^2>$ and by the amplifier self noise $<e_A^2>$. If the noise is Gauss-type it is possible to demonstrate that after amplification the Gaussian character is preserved.
ADS is the Analyzer for Statistical Distribution which gives the output as a function $P_{ca}(E)=P_{ca}[|u(t)|>E]$, u(t) being the instantaneous voltage at the input of the analyzer.

From the distribution plot one can obtain the variance σ_0^2 and then:

$$\langle e_0^2 \rangle = \frac{\sigma_0^2}{Y_0^2}$$

$$S_{eo}(f) = \frac{< e_0^2}{B_{eff}} = \frac{\sigma_0^2}{Y_0^2 \cdot B_{eff}} \qquad (6.7)$$

There are two possibilities to obtain the variance σ_0^2.

i. Operation at P_{ca}=constant.

For practical reasons one can select the value P_{ca}=0.31 at which the abscise is σ_t, the square root of the total input noise variance. It is necessary first to determine σ_A from a preliminary short-circuited input measurement, in the same conditions.
The two noise sources being un-correlated:

$$\sigma_t^2 = \sigma_A^2 + \sigma_0^2 \qquad (6.8)$$

Therefore σ_0^2, $<e_0^2>$ and $S_{eo}(f)$ will result.

ii. Operation at E=constant

The first operation mode has the disadvantage of precise gain requiring . This can be avoided if operates at $E=E_o$=constant

In this case for the amplifier own noise it is obtained P_{ca}^a and for total noise, P_{ca}^t (figure 6.3).

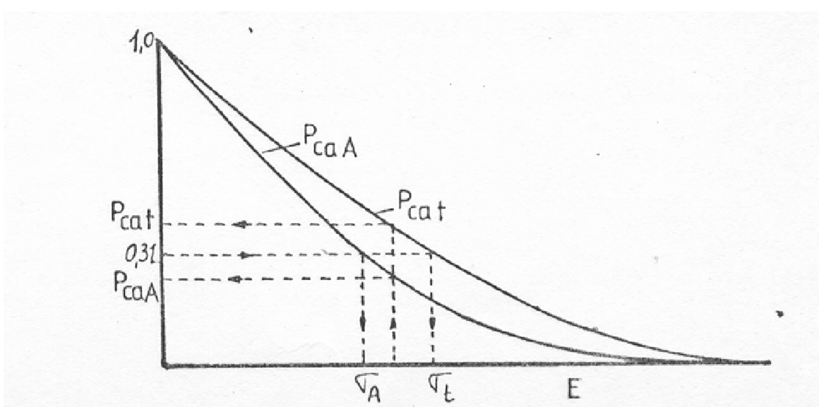


Figure 6.3 The variance obtained from Statistical Distribution

$$P_{ca}^t = erfc \frac{E_o}{\sigma_t \sqrt{2}} \qquad (6.9)$$

$$P_{ca}^{a} = erfc\frac{E_o}{\sigma_A\sqrt{2}}$$

Knowing Eo from a previous precise measurement, from tables result σ_A and σ_t for P_{ca}^{a} and P_{ca}^{t}.

The precision is better in this case.

In conclusion if there is a possibility to get experimentally the curve $P_{ca}(E)$ one can find σ being the abscissa corresponding to 0.317. Next $<e_0^2>$ and $S_{eo}(f)$ will be obtained.
In practice the full curve is not needed. Choosing two close values, E_1 and E_2 with $P_{ca}(E_1)<0.317< P_{ca}(E_2)$, σ results from linear interpolation.

One method to obtain this distribution can be based on the following equation:

$$P_{ca}(E) = \frac{\sum_{i=1}^{n}\Delta t_i}{T} \tag{6.10}$$

The sum represents the the total time in the interval T when the signal is higher in absolute value than E.

6.2 Narrow Band Noise

It is useful to obtain the frequency dependence of $S_{eo}(f)$.
If the Gaussian noise is filtered with a narrow pass-band filter with center frequency f_0, followed by a peak detector, the resulting signal is still noise but with a Rayleigh distribution (figure 6.4).

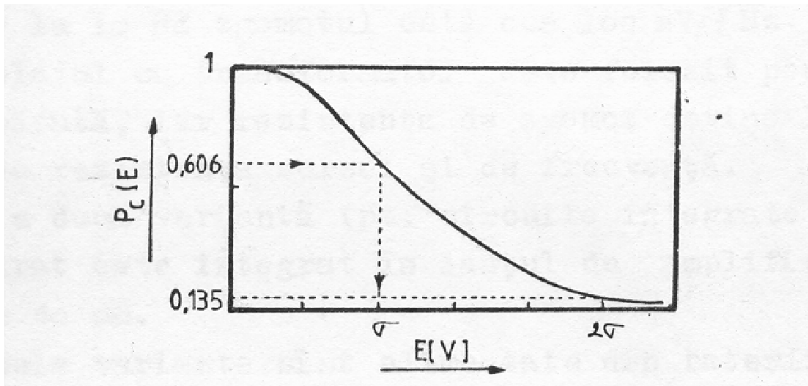


Figure 6.4 Rayleigh Noise Distribution Function

Here:

$$P_c(E) = \exp\left(-\frac{E^2}{2\upsilon^2}\right) \qquad (6.11)$$

As before:

$$P_{ca}(\sigma) = \exp\left(\frac{\sigma^2}{2\sigma^2}\right) = 0.606 \qquad (6.12)$$

Following the same procedure as before, $\sigma(f_0)$ and $S_{eo}(f_0)$ will result for P_{ca}=0.606.

